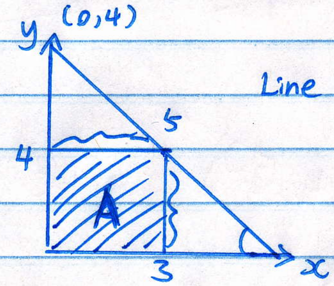
① Find the dimensions of the rectangle of maximum area that can be inscribed in a right triangle with sides of length 3 and 4 and hypotenuse of length 5, if two sides of the rectangle lie along the two sides of the triangle.

Ans



Line along the hypotenus has the following equation $y = -\frac{4}{3}x + 4$ (Here we are assuming that the base of the right angled triangle is along the x-axis and the length is along the y axis.

Choose a point (a,b) on the hypotenuse, then $b = -\frac{4}{3}a + 4$

Then the area of the rectangle is given by

$$f(a) = a\left(-\frac{4}{3}a + 4\right) = -\frac{4}{3}a^2 + 4a$$

Goal : want to maximize $f(a)$

$$f'(a) = -\frac{8a}{3} + 4, \quad f'(a) = 0 \Rightarrow -\frac{8}{3}a + 4 = 0 \Rightarrow \cancel{+}\frac{8a}{3} = \cancel{+}4$$

$$\Rightarrow 8a = 12 \Rightarrow a = \frac{12}{8} = \frac{3}{2}$$

Check that for $0 < a < \frac{3}{2}$, $f'(a) > 0$ and for $\frac{3}{2} < a < 3$, $f'(a) < 0$

Therefore by the FDT, $f(a)$ has a maximum at $a = \frac{3}{2}$ //